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AUTHOR(S):

Shikakura, Yosuke; Fukahata, Yukitoshi; Matsu'ura, Mitsuhiro

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**Spatial relationship between topography and rock uplift patterns in asymmetric
mountain ranges based on a stream erosion model**

Yosuke Shikakura^{a,*}, Yukitoshi Fukahata^b, and Mitsuhiro Matsu'ura^c

^a *Department of Geophysics, Graduate School of Science, Kyoto University,
Kitashirakawa Oiwake-cho, Sakyo-ku, Kyoto 606-8502, Japan*

^b *Disaster Prevention Research Institute, Kyoto University, Gokasho, Uji,
Kyoto 611-0011, Japan*

^c *Institute of Statistical Mathematics, Tachikawa, Tokyo 190-8562, Japan*

*Corresponding Author: Yosuke Shikakura

Tel: +81 75 753-3948

Fax: +81 75 753-3714

E-mail: shikakura@kugi.kyoto-u.ac.jp

Yukitoshi Fukahata

E-mail: fukahata@rcep.dpri.kyoto-u.ac.jp

Mitsuhiro Matsu'ura

E-mail: mat-lab@ism.ac.jp

Abstract

The spatial relationship between topography and rock uplift patterns in asymmetric mountain ranges was investigated using a stream erosion model in which the asymmetric rock uplift was given and erosion rates were proportional to the m -th power of the drainage area and the n -th power of the channel gradient. The model conditions were simple, and thus the effects of horizontal rock movement, diffusional processes, and erosion thresholds were neglected, and spatially uniform precipitation, lithology, and vegetation were assumed. In asymmetric mountain ranges, under realistic exponent conditions ($m < n$) and the above assumptions, the surface erosion rate is faster on the steeper side and slower on the gentler side. The topographic axis migrates away from the rock uplift axis toward the center of the mountain range owing to the contrast in erosion rates. This migration continues until the erosion is balanced with rock uplift. In a dynamic steady state, the topographic pattern is independent of the rock uplift rate as indicated by an analytical solution, and is prescribed by the rock uplift pattern and the exponents m and n . As the asymmetry of the rock uplift pattern increases, the topographic axis migrates a greater distance. The location of the topographic axis is related to the location of the rock uplift axis by a simple logarithmic function, for a wide range of m and n . The fit of the numerical results and the logarithmic function is particularly good when $m = 0.5$ and $n = 1.0$. If the rock uplift pattern in asymmetric mountain ranges is known, the value of $n - 5m/4$ can be constrained based on the logarithmic relation, assuming a dynamic steady state. On the other hand, if the value of $n - 5m/4$ is known in an asymmetric mountain range, the rock uplift pattern can be estimated directly from the topography. This relation was applied to the Suzuka Range in central Japan, and the value of $n - 5m/4$ was estimated for an assumed reverse fault motion.

Keywords

rock uplift, topography, stream erosion model, asymmetric range, logarithmic relation

1. Introduction

Remarkable recent developments in geodetic techniques have produced detailed topographic information for the entire world (e.g., Farr et al., 2007). Detailed elevation maps have even been obtained for some satellites and planets, such as the moon (Araki et al., 2009) and Mars (Smith et al., 1999). In contrast, the data of tectonic movements are very limited in space and time. For example, even Japan's GEONET, one of the densest GPS networks in the world, provides crustal movement data at a spacing of about 20 km for a time span of less than two decades (Sagiya, 2004). Data relevant to the long-term tectonic movements (10^3 – 10^6 years or more) that control topographic evolution are even sparser. Information of long-term tectonic deformation is fundamentally important to deduce tectonic processes. A principal goal of tectonic geomorphology would be to extract information regarding the rates and patterns of tectonic deformation directly from topography.

The purpose of this study is to quantitatively estimate rock uplift rate patterns using the topography of asymmetric mountain ranges. Asymmetry of mountain ranges may be caused by horizontal rock movements (Adams, 1980; Willett et al., 2001; Herman and Braun, 2006; Miller et al., 2007), differential base levels of erosion (Ellis and Densmore, 2006), asymmetric distribution of rain and snow precipitation (Beaumont et al., 1992; Willett, 1999; Mitchell and Montgomery, 2006; Anders et al., 2008), or differences in lithology (Adams, 1980). Asymmetric rock uplift, however, should also result in asymmetric mountain ranges (Koons, 1989; Kooi and Beaumont, 1996; Kühni and Pfiffner, 2001). For example, the Suzuka Range in central Japan has been differentially uplifted by a west-dipping reverse fault system at the eastern margin of the range (Ota and Sangawa, 1984), which results in asymmetric topography.

In this study, asymmetric vertical rock uplift is considered to be a proxy for tectonic movement. Although lateral advection may be important in some asymmetric ranges, incorporating this effect requires the addition of more model parameters, such as the dip angle and length of the fault. Furthermore, natural faults commonly have some curvature. If even more realistic models are sought, for example, the elastic thickness of

the lithosphere must also be taken into account (e.g., Thatcher and Rundle, 1984; Fukahata and Matsu'ura, 2006). Although realistic, such complicated modeling is not the aim of this study, which focuses on a simpler set of conditions.

To express surface erosion, a stream erosion model (e.g., Howard and Kerby, 1983; Howard et al., 1994; Tucker and Slingerland, 1994) is used, in which the long-term erosion rate is proportional to the product of power functions for the drainage area and channel gradient. The values of the exponents of the power functions have been estimated in some mountain ranges from observed incision rates along river profiles (Stock and Montgomery, 1999; Whipple et al., 2000; van der Beek and Bishop, 2003; Harkins et al., 2007) and from the intrinsic concavity index of rivers with some assumptions of rock uplift rates under steady state conditions (e.g., Tarboton et al., 1989; Seidl and Dietrich, 1992; Kirby and Whipple, 2001; Meade, 2010). However, the relation between the values of the exponents and asymmetry of topography has not been elucidated.

The response of topography to asymmetric (gabled) vertical rock uplift is examined using the stream erosion model. The location of main drainage divides (topographic axes) is of particular importance because these divides determine the first-order topographic features and drainage patterns in mountain ranges, which prescribe the environment and landscape of the area. If there is no erosion, the topographic axis must simply coincide with the rock uplift axis. Under asymmetric rock uplift and fluvial erosion, however, the erosion rate is seemingly faster on the steeper side, as long as all other conditions that contribute to erosion rate, such as precipitation, lithology, and vegetation, are uniform. Asymmetric uplift and stream erosion would predict, therefore, that the topographic axis migrates away from the rock uplift axis toward the center of the mountain range. The aim of this study is to systematically quantify the distance between the topographic and rock uplift axes. Because the distance also depends on the exponent values of the stream erosion model, this study links to estimation of these values.

This study employs a topographic evolution model that incorporates the effect of

rock uplift into a stream erosion model. Under conditions of asymmetric rock uplift, topographic evolution is numerically simulated, and a curious relationship between topographic and rock uplift axes is shown. We then investigate the dependence of the relation on the values of the exponents, and discuss the implications of the results for real mountain ranges.

2. Topographic evolution model

The stream erosion model is suitable for modeling fluvial erosion at a resolution of 1 km and was developed based on the law of open channel flow for stream incision into bedrock (e.g., Howard and Kerby, 1983; Howard et al., 1994; Tucker and Slingerland, 1994). In the stream erosion model, the erosion rate ε is expressed as the product of power functions for the drainage area A and channel gradient S as follows:

$$\varepsilon(x, y, t) = K [A(x, y, t)]^m [S(x, y, t)]^n \quad (1)$$

where K is the coefficient of erosion and the dimensionless exponents m and n are positive constants related to basin hydrology, hydraulic geometry, and erosion process (Howard et al., 1994; Whipple and Tucker, 1999; Whipple et al., 2000). In the present study, surface deposition (e.g., Willgoose et al., 1991; Kooi and Beaumont, 1994; Tucker and Whipple, 2002), and hillslope processes (e.g., Schmidt and Montgomery, 1995; Roering et al., 1999; Stark and Hovius, 2001; Anderson, 2002) were neglected, and the erosion threshold that depends on the critical shear stress (e.g., Howard et al., 1994; Whipple and Tucker, 1999; Tucker and Bras, 2000; Lague et al., 2005) was not employed, for simplicity. Other factors that contribute to erosion rate, such as precipitation, lithology, and vegetation, are assumed to be spatially and temporally uniform.

Rock uplift u is incorporated into the stream erosion model of Eq. (1) as in many

previous studies (e.g., Willgoose et al., 1991; Kirkby, 1994; Tucker and Slingerland, 1994; Kirby and Whipple, 2001; Miller et al., 2007) as

$$\frac{d}{dt}h(x, y, t) = u(x, y, t) - K[A(x, y, t)]^m [S(x, y, t)]^n \quad (2)$$

where h represents altitude. The drainage area A and the channel gradient S are functions of h . In other words, if a topographic distribution $h(x, y, t)$ is given, $A(x, y, t)$ and $S(x, y, t)$ are uniquely determined. Parameter S is the gradient of h . A is dependent on h in a complex fashion, but a proportional change in h from $h(x, y)$ to $ch(x, y)$, where c is an arbitrary positive constant, does not affect A .

Given the rock uplift rate $u(x, y, t)$, the initial height $h(x, y, 0)$, and the boundary conditions, Eq. (2) can be solved with a finite difference method. Following the scheme developed by Beaumont et al. (1992), the model can be discretized in space and time. A series of regular square poles is used to represent the discretized topography. The drainage area and channel gradient for each cell is computed using a steepest-descent flow accumulation algorithm (Tucker and Slingerland, 1994). In computing the drainage area for a particular cell, the area of that cell is included. If the cell concerned is not included, the drainage divide is fixed and can not migrate because the erosion rate at the headwater cell of each stream is always zero as there is zero drainage area.

In the numerical simulation, the model area is 100×100 km, which is divided into 1×1 km cells. The x -axis is the east direction, and the y -axis is the north direction. The origin $(x, y) = (0, 0)$ is at the center of the model area.

The asymmetric rock uplift rate in the x direction is given by the following function:

$$u(x, y, t) = \begin{cases} \frac{x+L}{x_u+L} u_{\max} & (-L \leq x \leq x_u) \\ \frac{L-x}{L-x_u} u_{\max} & (x_u \leq x \leq L) \end{cases} \quad (3)$$

where L is half of the model length ($L = 50$ km). The functional form is shown in Fig. 1. x_u represents the location of the rock uplift axis, and u_{\max} represents the maximum rock uplift rate, which is equal to the rock uplift rate at the rock uplift axis x_u . The rock uplift rate is uniform in the y direction and constant over time.

[Insert Fig. 1]

The initial conditions are a flat topography at an elevation of 0 m, with white noise with maximum amplitude of 10 m. The altitudes of the eastern and western boundaries are exactly set at 0 m through the computation. The water drains only through the eastern and western boundaries, and the northern and southern sides are no-flow boundaries. The problems associated with closed depressions (e.g., Tucker et al., 2001) are not included because the gradient of slopes generated by the following computations is usually very steep, except for just after initiation of rock uplift. For closed depressions, the erosion rate is set to zero.

3. Results

The relationship between rock uplift and topography was quantitatively investigated with the topographic evolution model introduced above. Different values for parameters x_u and u_{\max} , which prescribe the rock uplift rate, were used in order to determine the dependence of topography on these parameters. In contrast, parameters m , n , and K , which control the erosion rate, were initially fixed at $m = 0.5$, $n = 1.0$, and $K = 1.2 \times 10^{-5} \text{ yr}^{-1}$. These values imply that the stream incision rate was modeled as a function of stream power per unit bed area (e.g., Whipple and Tucker,

1999). As discussed in Section 3.4, the dependence of topography on these parameters was also examined.

3.1 Topographic evolution under asymmetric rock uplift

The rock uplift parameters x_u and u_{\max} were initially set at 30 km and 6 mm yr⁻¹, respectively, to observe the topographic evolution process (Fig. 2). The results show that after initiation of rock uplift, the mountain grows at approximately the same rate as uplift because erosion is very weak. The topographic axis, defined here as the line of the highest mean altitude along the north-south profiles, coincides almost exactly with the rock uplift axis. However, as the mountain grows, surface erosion increases, especially on the side of the range having the steepest topographic gradient (east). The topographic axis thus gradually migrates toward the center of the mountain range (Fig. 2). This means that the location of the topographic axis is more strongly affected by the effect of the channel gradient S than by the drainage area A , which is larger on the less steep side. A dynamic steady state is reached before $t = 4.0$ My. A dynamic steady state is defined as the condition under which the elevation change over a 1 My interval, averaged over the model area and normalized by the maximum elevation, is less than 0.01%. Before reaching a steady state, the topographic axis migrates about 10 km from the rock uplift axis toward the center. The elevation of the topographic axis is slightly reduced during this migration.

[Insert Fig. 2]

3.2 Relationship between rock uplift and topography

The relation of asymmetric rock uplift to topographic evolution was simulated by changing the maximum uplift rate u_{\max} and the location of the rock uplift axis x_u . Values of 2, 6, and 10 mm yr⁻¹ were used for u_{\max} , and x_u was taken every 5 km from 5 to 45 km for each u_{\max} . To suppress the dependence of topographic evolution on the initial random noise, the numerical simulation was performed five times for

each u_{\max} and x_u pair by changing the random noise of the initial topography. Fig. 3 shows the relation between u_{\max} and the location of the topographic axis in the steady state x_t for various values of x_u . As shown in the figure, x_t does not depend on u_{\max} irrespective of x_u . In the simulation of topographic evolution based on Eq. (2), the pattern of topography in the steady state is generally independent of the rate of rock uplift for the following reason. In the steady state, the left side of Eq. (2) is zero:

$$u(x, y) = K[A(x, y)]^m[S(x, y)]^n \quad (4)$$

Therefore, if a topographic distribution $h(x, y)$ is a solution of Eq. (4) under a given uplift rate $u(x, y)$, $\sqrt[n]{c}h(x, y)$ is also a solution for another given uplift rate with the same pattern $cu(x, y)$, where c is an arbitrary positive constant. Recall that a proportional change in height distribution from $h(x, y)$ to $c'h(x, y)$ does not affect the drainage area $A(x, y)$; only the channel gradient changes from $S(x, y)$ to $c'S(x, y)$. The minor fluctuation of each line in Fig. 3 is ascribed to the given random noise added to the initial topography. In brief, a proportional change in u from $u(x, y)$ to $cu(x, y)$ results in no change in A and a proportional change in S from $S(x, y)$ to $\sqrt[n]{c}S(x, y)$ in the steady state. Here, it should be noted that u , K and m may be spatially variable, but n must be constant in space.

[Insert Fig. 3]

From the theoretical relations described above, the height of the topographic axis in the steady state should be proportional to the rock uplift rate to the $1/n$ power. Given that n is taken to be unity, we show the elevation of the topographic axis in the steady state h_{\max} normalized by u_{\max} as a function of x_u in Fig. 4A. Each line of different rock uplift rates overlaps almost completely with each other. This means that h_{\max} is indeed proportional to rock uplift rate. In addition, h_{\max} decreases with x_u , which is consistent with the topographic evolution process shown in Fig. 2, where the elevation

of the topographic axis decreases as the topographic axis moves away from the rock uplift axis x_u toward the center. The distance between x_u and x_t increases with x_u , as shown in Fig. 3 and Fig. 5.

[Insert Fig. 4]

Fig. 4B shows the time required to reach a dynamic steady state for each x_u and u_{\max} pair. As expected, the time to reach a steady state is longer for larger x_u . The topographic axis slowly moves from the rock uplift axis toward the center of the mountain range (Fig. 2), and the distance increases with x_u (Fig. 3). During this time, the drainage network continues to evolve and adapt. This process takes time. Comparison of Fig. 4A and B shows that the time required is most sensitive to the initial random noise.

The time required is also independent of the maximum rock uplift rate u_{\max} (Fig. 4B). For this independence, the key is that n is taken to be unity. When a height distribution $h(x, y, t)$ is a solution of Eq. (2) for a given uplift rate $u(x, y, t)$ that is constant in time, the height distribution $ch(x, y, t)$ is also a solution for another given uplift rate with the same pattern $cu(x, y, t)$. Recall again the drainage area $A(x, y, t)$ is constant for a proportional change in height distribution from $h(x, y, t)$ to $ch(x, y, t)$ and that $S(x, y, t)$, which is the gradient of $h(x, y, t)$, is proportional to $h(x, y, t)$. In brief, the proportional relation of topography to rock uplift rates holds true throughout the time development. However, it should be noted that the proportional relation does not hold true when $n \neq 1$; only in the steady state, the proportional relation of topography to the n -th root of the rock uplift rate (Fig. 3 and Fig. 4A) holds true for $n \neq 1$. The independence of the elapsed time relative to the magnitude of rock uplift rate has already been noted by Kooi and Beaumont (1996) for the case of symmetric rock uplift. The independence is also valid for asymmetric rock uplift.

3.3 Logarithmic relation between topographic and rock uplift axes

The relation between the rock uplift axis x_u and the topographic axis x_t in the steady state (Fig. 3) requires examination. Any proportional change in a given rock uplift rate, from $u(x, y)$ to $cu(x, y)$, does not affect the location of the topographic axis under steady-state conditions (and for transient behavior when $n=1$). The numerical simulation was run five times for each u_{\max} and x_u pair, and the average and standard deviation of 15 trials with different rock uplift rates were assessed as a group.

The results show that x_t increases with x_u (Fig. 5), but the rate of increase in x_t is not as rapid as that in x_u . The numerical relation between x_t and x_u needs to be expressed with an analytical function. The function must increase monotonically and the differentiation of the function should decrease monotonically; therefore, a logarithmic function would be appropriate. As a boundary condition, the function must pass through the origin ($x_u = x_t = 0$) because there is no need for the topographic axis to move away from the center of the model region when x_u equals zero. In addition, a very small change in x_u from zero seems to cause the same amount of change in x_t . This condition is written as $dx_t / dx_u = 1$ at $x_u = 0$. The logarithmic function to fit the numerical relation is then expressed by

$$x_t = f(x_u) = a \ln \left(\frac{x_u}{a} + 1 \right) \quad (5)$$

A natural logarithmic function is used here, but even if an apparently different base is used for the logarithmic function, the functional form is the same as Eq. (5) after transformation of the base. Eq. (5) consists of only the fitting parameter a , which is determined using the least-square method.

[Insert Fig. 5]

The fit of the logarithmic function to the numerical results is statistically significant

(Fig. 5). The function passes almost exactly through the average of each trial. The average misfit to the nine data points is less than 0.2 km, which is much smaller than the grid interval of the numerical simulation. It should be noted that the result of each trial does not necessarily fit the logarithmic function, as shown by the error bar representing the standard deviation. After many trials with different initial random noise, however, the averages of the results follow the natural logarithmic function.

So far, parameters m , n , and K , which control erosion rates, have been fixed. Parameter K does not affect the functional relation between x_t and x_u because K has the same effect (except for the reciprocal) as u (Eq. (4)). However, x_t does depend on m and n . The effect of the exponents m and n on the logarithmic relation between x_t and x_u requires investigation.

3.4 Dependence of the logarithmic relation on the exponents

As explained by Tucker and Whipple (2002), Eq. (1) has been used to model various fluvial erosion processes, including bed shear stress ($m \simeq 0.3, n \simeq 0.7$) (Howard and Kerby, 1983; Howard et al., 1994; Tucker and Slingerland, 1997), stream power per unit channel length ($m \simeq n = 1.0$) (Seidl and Dietrich, 1992), and stream power per unit bed area ($m \simeq 0.5, n \simeq 1.0$) (Stock and Montgomery, 1999; Whipple and Tucker, 1999; Kirby and Whipple, 2001). They also pointed out that the computed stream profile concavity under the condition $m/n \geq 1.0$ or $m/n < 0.2$ is not common for most mountain drainage basins.

The values of the exponents m and n as well as those of x_u and u_{\max} are changed: n is set at 0.5, 1.0, and 1.5, and m/n has values that increase in increments of 0.1, from 0.3 to 0.8, for each value of n . For each pair of m and n , the same computation as in Fig. 5 is repeated, and the relationship between the location of the topographic axis in the steady state x_t and that of the rock uplift axis x_u for each pair of exponents m and n is plotted (Fig. 6). The fit of the logarithmic function is again very good for most cases, indicating that the value of the fitting parameter a can be determined from one datum point of x_t and x_u , regardless of the values of the

exponents m and n . The fit appears to be best when $m/n = 0.5$. For smaller m/n , the numerical results appear to be slightly more rectilinear than the logarithmic function, as for example in the case of $m/n = 0.3$. For larger m/n , the fit is still good. This may be due to the characteristics of the fitting function. The fitting function asymptotically approaches $x_t = x_u$ as m/n increases (Fig. 6). Therefore, even if the theoretical fitting function deviated from the logarithmic function, the deviation would not be noticeable.

[Insert Fig. 6]

The change in the fitting function in Fig. 6 corresponds to the change in the fitting parameter a in Eq. (5). For larger a , the fitting function approaches $x_t = x_u$, but for smaller a , the curvature of the fitting function becomes more acute. The value of a is controlled by the relative susceptibility of drainage area A and channel gradient S to erosion. The relative contribution of the drainage area, which is larger on the gentler side, increases with larger m/n . This means that for larger m/n , the topographic axis x_t does not move away from the rock uplift axis x_u as much, the fitting function approaches $x_t = x_u$, and a becomes large. In contrast, for smaller m/n , the topographic axis migrates a substantial distance, the curvature of the fitting function is more acute, and the fitting parameter a becomes smaller. As shown in Fig. 6, however, the value of a can vary significantly for smaller m/n , even if m/n has the same value. In short, parameter a is not dependent only on m/n .

A contour map of parameter a on the m and n coordinate plane (Fig. 7A) shows that a is roughly constant for $n - pm$, where p is about $5/4$. If a is then plotted relative to $n - pm + q$ on a double logarithmic diagram (Fig. 7B) where q is a constant parameter, the numerical results can be fitted by a linear function expressed as

$$\log_{10} a = r \log_{10} (n - pm + q) + s \quad (6)$$

where the optimal parameter values determined with the least-square method are $p = 1.26$, $q = 0.415$, $r = -2.55$, and $s = 1.11$. The fit is again excellent. If $p = 1.25$ is used instead of $p = 1.26$, the fit is nearly the same. This means that $n - 5m/4$ is an appropriate indicator of the relative susceptibility of drainage area A and channel gradient S to erosion. For smaller $n - 5m/4$, in which the contribution of the drainage area is relatively high, the value of a becomes larger, and vice versa. With Eqs. (5) and (6), the relation between x_t and $n - 5m/4$ are obtained for various values of x_u (Fig. 8). This diagram means that we can estimate $n - 5m/4$ or x_u , if we know either value of them, because the topography (i.e., x_t) is commonly well known.

[Insert Fig. 7], [Insert Fig. 8]

4. Discussion

As shown in the numerical simulations, the erosion rate in asymmetric mountain ranges is faster on the steeper side and slower on the gentler side under realistic exponent conditions ($m < n$), as long as the other conditions that contribute to erosion rate are uniform. For example, in the Taiwan Central Range, where the eastern slope is much steeper than the western slope, significantly faster erosion is suggested for the eastern flank as compared to the western flank, as indicated by the metamorphic grades of exposed rocks (Ho, 1986), fission track analyses (Willett et al., 2003), and heat-flow data (Yamano, 1995; Fukahata and Matsu'ura, 2001).

To address the distance migrated by the topographic axis relative to the rock uplift axis, the model was applied to the Suzuka Range in central Japan as an example (Fig. 9A). The Suzuka Range is located in the Kinki Triangle (Huzita, 1962), an area currently under east-west compression (Huzita, 1980; Terakawa and Matsu'ura, 2010). The eastern limit of the Suzuka Range is bounded by west-dipping reverse faults of the Ichishi Fault system, which have been active since the late Pliocene (Ota and Sangawa, 1984; Ishiyama et al., 1999). There is no significant local variation in lithology (Huzita, 1962; Geological Survey of Japan, AIST, 2009) and precipitation across the range.

[Insert Fig. 9]

A cross-section of the topographic profile of the range (Fig. 9B) was produced by averaging the elevation data in the direction parallel to the trend of the range (approximately north-south, Fig. 9A). In the topographic profile, the horizontal distance and relative elevation of the mountain range are normalized by the width of the range L_R (20 km) and the difference between the maximum (812 m) and minimum (170 m) heights. The topographic axis of the Suzuka Range is located at about 0.20 (normalized distance) as shown in Fig. 9B.

The uplift and erosion rates across the Suzuka Range are not known. A seismic reflection survey showed that the Fumotomura Fault, one of the faults of the Ichishi Fault system, has a dip angle of about 60° at shallow depth (Ishiyama et al., 1999). The youngest strata, the Tokai Group, are cross-cut by the fault, but the displacement is very small. West of the fault, strata of the Tokai Group steeply dip eastward, indicating that deformation around the eastern end of the range is caused by a fault-propagation fold (Ishiyama et al., 1999). Based on the seismic survey, we simulated the crustal deformation pattern in the Suzuka Range. The fault was assumed to have a dip angle of 60° at the earth's surface, which gradually changes to a horizontal detachment fault at depth (Fig. 10A). The slip rate along the fault was assumed to decrease gradually to zero at surface from 3 km at depth because the deformation is caused by a fault-propagation fold. It should be noted that the true fault geometry and slip rate distribution are not well determined from observations; the configuration described was adopted to demonstrate the use of the model described in the previous section. In the computation of crustal deformation due to the dislocation across the fault plane, an elastic half-space was assumed and a value of 50 GPa was used for the bulk modulus and 30 GPa for the shear modulus.

[Insert Fig. 10]

Fig. 10B and C shows the profiles of vertical and horizontal displacements, respectively. The rock uplift axis is located at 0.32 in normalized distance (Fig. 10B). When $x_u = 0.32$ and $x_t = 0.20$, we can obtain the estimate of $n - 5m/4$ to be 0.38 ± 0.05 based on Fig. 8, where the estimation error is originated from the standard deviation shown in Fig. 6. $n - 5m/4 = 0.38$ almost coincides with a typically used set of the exponent values of $(m, n) = (0.5, 1.0)$. Then, $n - 5m/4$ is 0.375. If the topography of the Suzuka Range is before the dynamic steady state, the value of $n - 5m/4$ becomes larger, because x_t further moves toward the center of the range.

So far, only the purely gabled rock uplift pattern has been considered as a cause of topographic asymmetry. In actual mountain ranges, however, the other factors may significantly affect the relation between the rock uplift axis x_u and the topographic axis x_t . So, we carried out some sensitivity tests, in which $x_u = 0.32$ and $(m, n) = (0.5, 1.0)$ were used, and the topographic evolution was computed 15 times for each setting with different initial random noise.

As shown in Fig. 10B and C, the computed uplift pattern is not purely gabled and the horizontal displacement is not zero. Thus, the computed displacement profiles (solid lines in Fig. 10B and C) were used in the numerical simulation of topographic evolution, instead of the purely gabled rock uplift pattern (dotted line in Fig. 10B). As a result, 0.217 ± 0.007 was obtained for x_t . This value is close to 0.207 ± 0.010 , which is obtained for the purely gabled rock uplift pattern. If the horizontal displacement is neglected and only the computed uplift profile (solid line in Fig. 10B) is used, 0.225 ± 0.012 is obtained. The effect of horizontal displacement is not significant in this situation, because the horizontal displacement in the hanging wall is quite uniform (Fig. 10C). The effect of asymmetrical precipitation was also investigated by giving 20 % larger precipitation (larger A in the simulation) in one side. The topographic axis at each time step was used as the boundary of the different precipitation. Then, x_t is 0.179 ± 0.011 and 0.227 ± 0.007 for larger precipitation in the eastern and western sides, respectively. When the base level of erosion was changed 10 % in the normalized

height, x_l was 0.189 ± 0.014 and 0.223 ± 0.010 for the higher base level in the western and eastern sides, respectively. In either case, the calculated x_l is almost within the two sigma of the purely gabled uplift case demonstrated in the previous section.

In the numerical simulation, the effects of thresholds for erosion and hillslope processes were omitted for simplicity. Owing to this simplification, the numerical model is essentially described by only three parameters, the rock uplift axis x_u and the exponents m and n , and does not depend on the rock uplift rate (Fig. 3 and Fig. 4) or the coefficient of erosion K . Numerical experiments in which a threshold of critical shear stress for stream erosion is included do not cause notable changes, probably due to a rapid erosion rate in the numerical simulation. Although the model does not explicitly incorporate hillslope processes, the uppermost cells of streams can be eroded because the area of any given cell is included in the calculation of the drainage area. This differs from the models of Kooi and Beaumont (1996) and Kühni and Pfiffner (2001). If we incorporate hillslope processes into the model, the migration distance of the topographic axis would probably be larger because the second derivative of the topography $\nabla^2 h$ would be reduced. The main features of the topography, however, are considered to be primarily controlled by an advective stream process (Howard, 1994; Kooi and Beaumont, 1996). In fact, the numerical simulation of Miller et al. (2007), in which the scale of their target (Siwalik Hills, Nepal) is about 20 km in the horizontal direction, demonstrated that diffusion does not visibly affect the asymmetry of the range under normal conditions.

This study identifies the logarithmic relation between the topographic and rock uplift axes for a simple set of conditions (Fig. 5 and Fig. 6). The fitting parameter a of the logarithmic function was also clearly related to the exponents m and n (Fig. 7). To reveal the nature of the logarithmic relation, we tried to derive these relations analytically, but failed despite of its simplicity. This problem should be addressed in the future.

5. Conclusions

Asymmetric rock uplift is one of the main causes of asymmetric mountain ranges, such as the Suzuka Range in central Japan. In asymmetric mountain ranges, the surface erosion rate is faster on the steeper side and slower on the gentler side, under realistic conditions for the exponents ($m < n$) and assuming that the other conditions that control the erosion rate, such as precipitation, lithology, and vegetation, are uniform. Therefore, the topographic axis migrates from the rock uplift axis toward the center of the mountain range until erosion is balanced by rock uplift (Fig. 2); this balancing takes a few million years (Fig. 4).

The migration distance from the rock uplift axis can be expressed by a logarithmic function (Fig. 6). The fit of the logarithmic function is especially good when $(m, n) = (0.5, 1.0)$ (Fig. 5). A proportional change in the uplift rate does not affect this relationship (Fig. 3 and Fig. 4) because a change in the uplift rate from $u(x, y)$ to $cu(x, y)$ results in no change in the drainage area A and a proportional change in S from $S(x, y)$ to $\sqrt[n]{c}S(x, y)$. Here, u , K , and m may be spatially variable, but n must be constant.

If the rock uplift pattern for an asymmetric mountain range is similar to be gabled, the value of $n - 5m/4$ can be constrained (Fig. 8). This concept was applied to the Suzuka Range in central Japan (Fig. 9 and Fig. 10). Conversely, if a value of $n - 5m/4$ is available, the rock uplift pattern can be estimated directly from the topography in asymmetric mountain ranges.

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Figure Captions

Fig. 1. Profile of asymmetric rock uplift rate in the x (east-west) direction, equivalent to Eq. (3). x_u and u_{\max} represent the location of the rock uplift axis and the maximum rock uplift rate, respectively. The rock uplift rate is uniform in the y (north-south) direction and constant over time.

Fig. 2. Topographic evolution process under a given asymmetric rock uplift rate. The initial topography is flat with an elevation of 0 m and superimposed white-noise topography with a maximum amplitude of 10 m. (A) Snapshots of topography over 4 My of elapsed time, with a contour interval of 1000 m. (B) East-west topographic profiles averaged in the y (north-south) direction. x_t and h_{\max} are the location and elevation of the topographic axis in the steady state, respectively. The rock uplift axis x_u , represented by the thick broken line in each diagram, is 30 km, and the maximum uplift rate u_{\max} is 6 mm yr⁻¹. Water is drained only from the western (left) and eastern (right) boundaries.

Fig. 3. Relation of the location of the topographic axis in the steady state x_t to the maximum rock uplift rate u_{\max} for various rock uplift axes x_u . To suppress the effect of the initial random noise, the numerical simulation was performed five times for each pair of u_{\max} and x_u . The solid diamond and error bar represent the average and standard deviation of the five trials, respectively.

Fig. 4. Relation between parameters of rock uplift and topography. (A) Relation of the height h_{\max} at the topographic axis in the steady state to x_u and u_{\max} . In order to compare results, the height at the topographic axis is normalized by u_{\max} . (B) The relation of the elapsed time to steady state versus x_u and u_{\max} . The solid diamond and error bar in each diagram represent the average and standard deviation of the five trials, respectively, for each pair of x_u and u_{\max} .

Fig. 5. Relation of the location of the topographic axis in the steady state x_t to the location of the rock uplift axis x_u . The numerical results are fitted by a natural logarithmic function in Eq. (5). In this figure, parameter a is 22.2 km. The solid

diamond and error bar represent the average and standard deviation of 15 trials, respectively. For reference, $x_t = x_u$ is also shown (dotted line).

Fig. 6. Relation between x_t and x_u for various pairs of the exponents m and n . The solid diamond and error bar represent the average and standard deviation of 15 trials, respectively, for the numerical simulation with different random noise of initial topography. The fitted logarithmic function for each pair of exponents has the form of Eq. (5), and the fitting parameter a is determined by the least-square method. For reference, $x_t = x_u$ is also shown (dotted line).

Fig. 7. Relation of the log-fitting parameter a (Eq. 5) with the exponents m and n . (A) Contour map of parameter a on the m and n coordinate plane. (B) Relation between parameter a and a linear function of m and n on a double logarithmic diagram. The solid line represents the fitting function given in Eq. (6).

Fig. 8. Relation of the location of the topographic axis in the steady state x_t to an appropriate indicator of the susceptibility of erosion $n - 5m/4$ for various rock uplift axes x_u .

Fig. 9. Topography of the Suzuka Range in central Japan. (A) Location map of the Suzuka Range. The rectangle indicates the region where the elevation data were averaged for the topographic cross-section in Fig. 9B. The area of the Kinki triangle is also shown by dotted lines (Research Group for Active Faults of Japan, 1991). The inset shows the tectonic setting of this area; EUR, NAM, and PHS stand for the Eurasian, North American, and Philippine Sea plates, respectively. (B) Topographic cross-section for the Suzuka Range. The cross-section is calculated from the digital map of Japan (Geographical Survey Institute of Japan, 1997) by averaging the elevation data in the direction parallel to the trend of the range. The horizontal distance and relative elevation are normalized by the width of the range L_R and the difference between the maximum and minimum elevations $h_{\max} - h_{\min}$, respectively. L_R , h_{\max} , and h_{\min} are about 20 km, 812 m, and 170 m, respectively.

Fig. 10. Crustal deformation pattern caused by reverse fault movement. (A) Vertical cross-section of the assumed fault geometry. The slip rate gradually decreases from

713 3 km at depth to zero at the earth's surface. (B) Uplift rate profile at the earth's
714 surface due to the fault motion (solid line), normalized by the maximum rock uplift
715 rate. Rock uplift pattern in the form of Eq. (3) with $x_u = 32\text{km}$ is also shown
716 (broken line). (C) Horizontal displacement profile at the earth's surface due to the
717 fault motion, normalized by the maximum rock uplift rate.

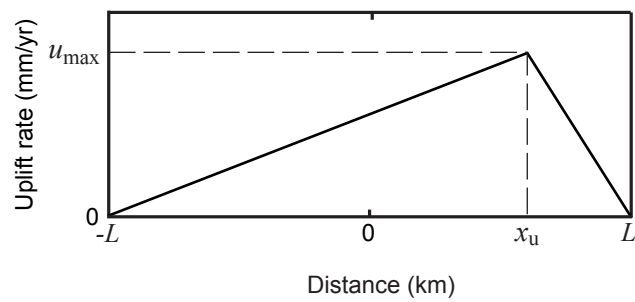


Fig. 1

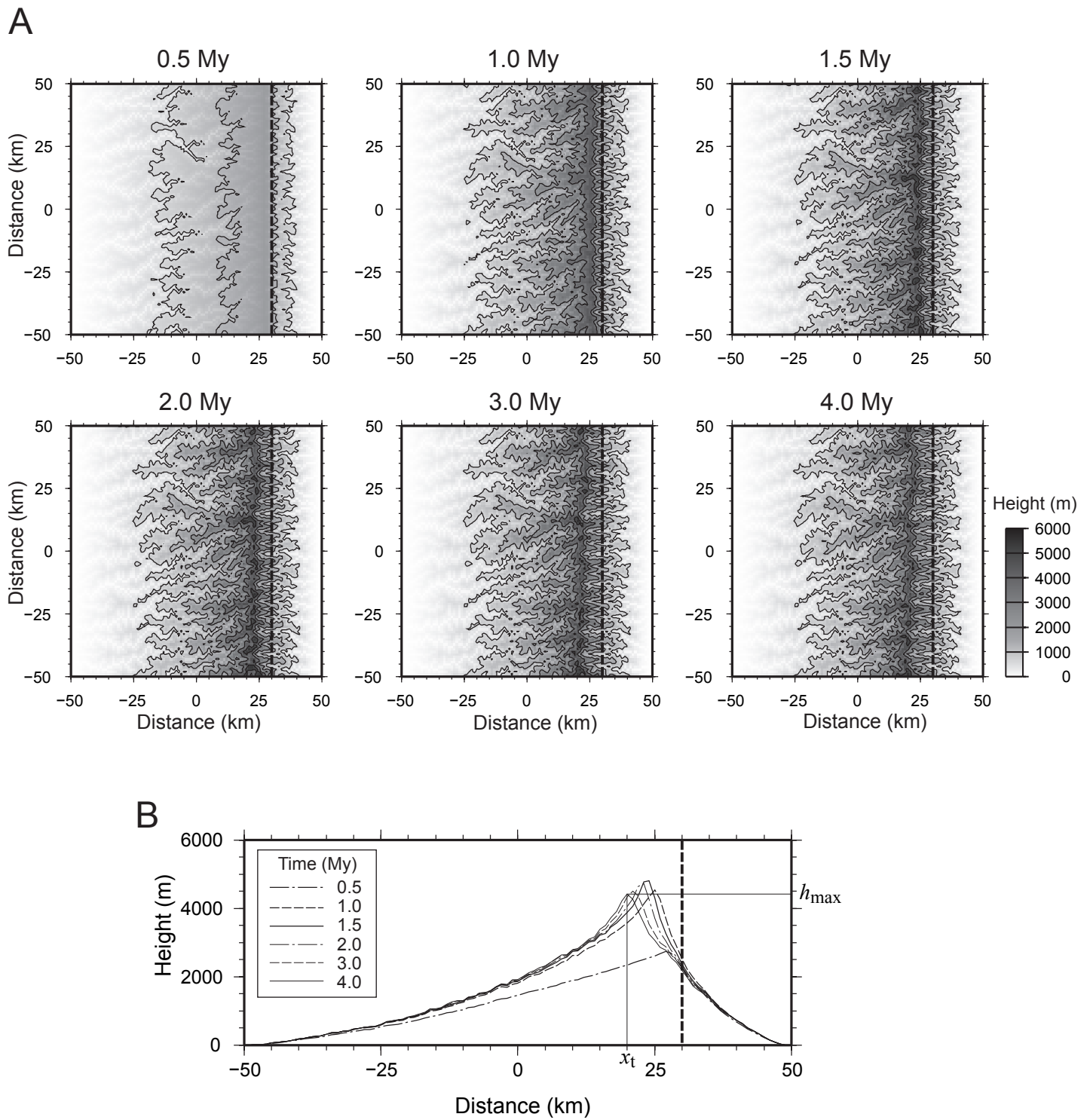


Fig. 2

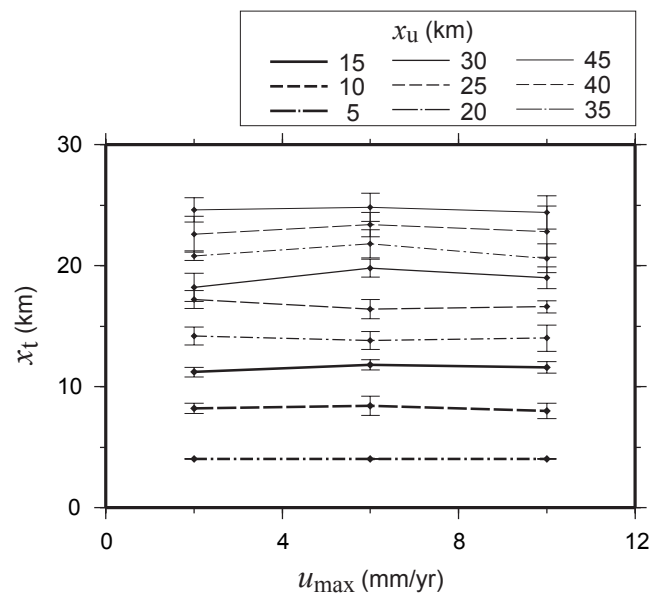


Fig. 3

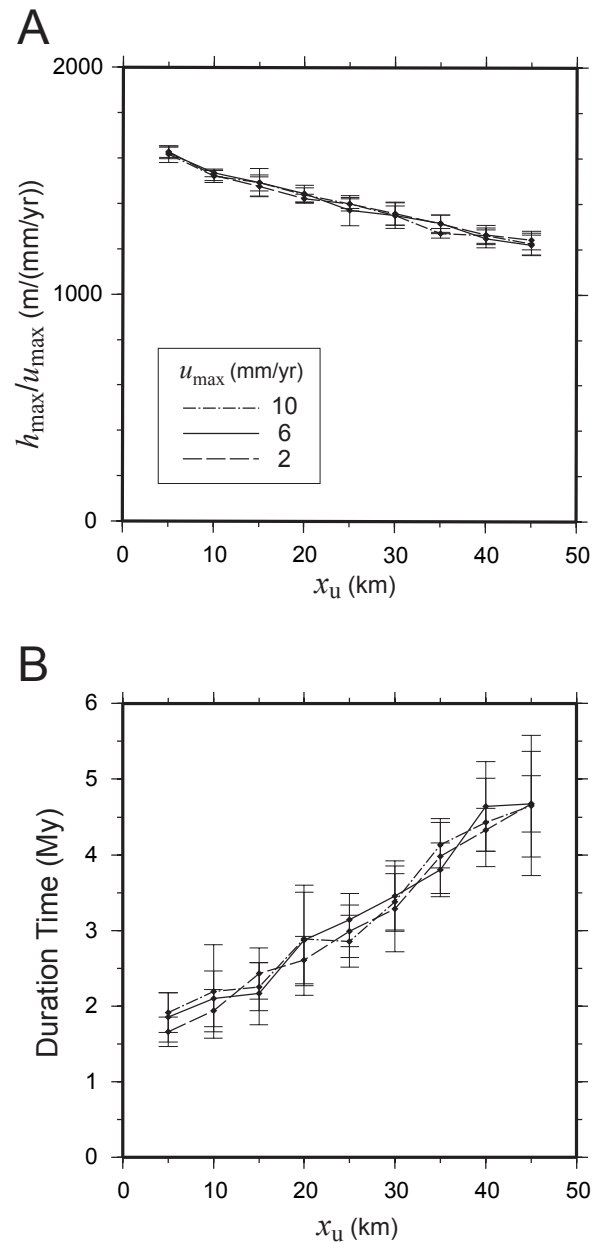


Fig. 4

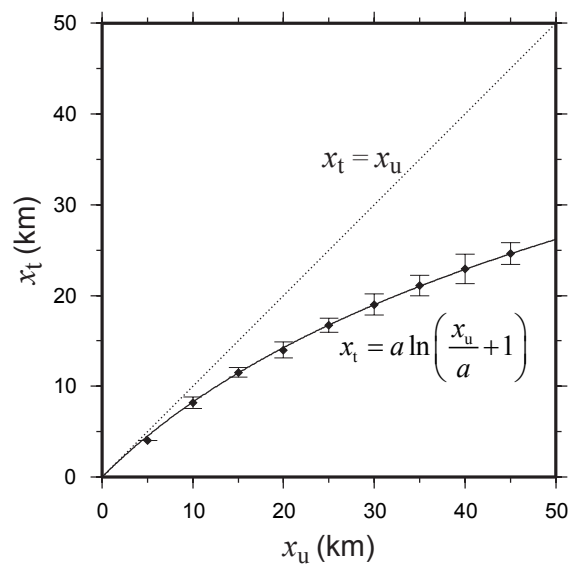


Fig. 5

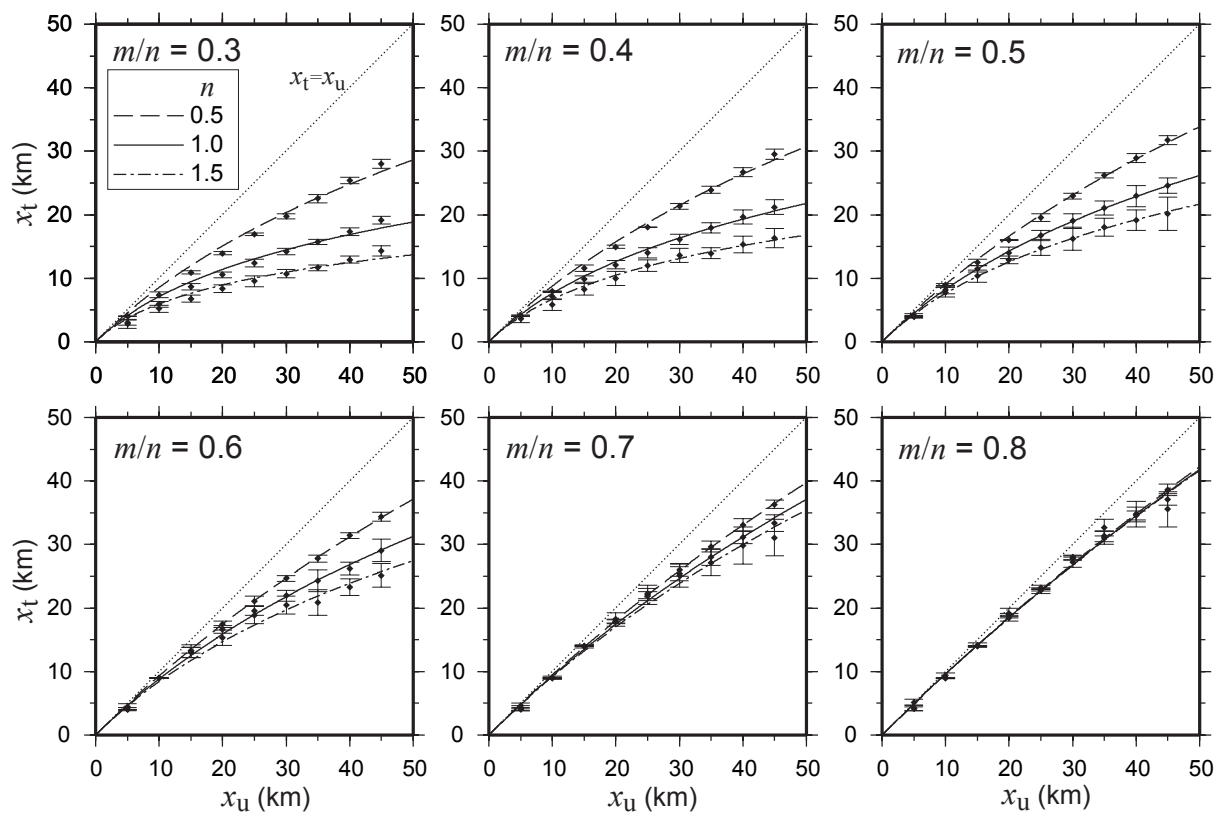


Fig. 6

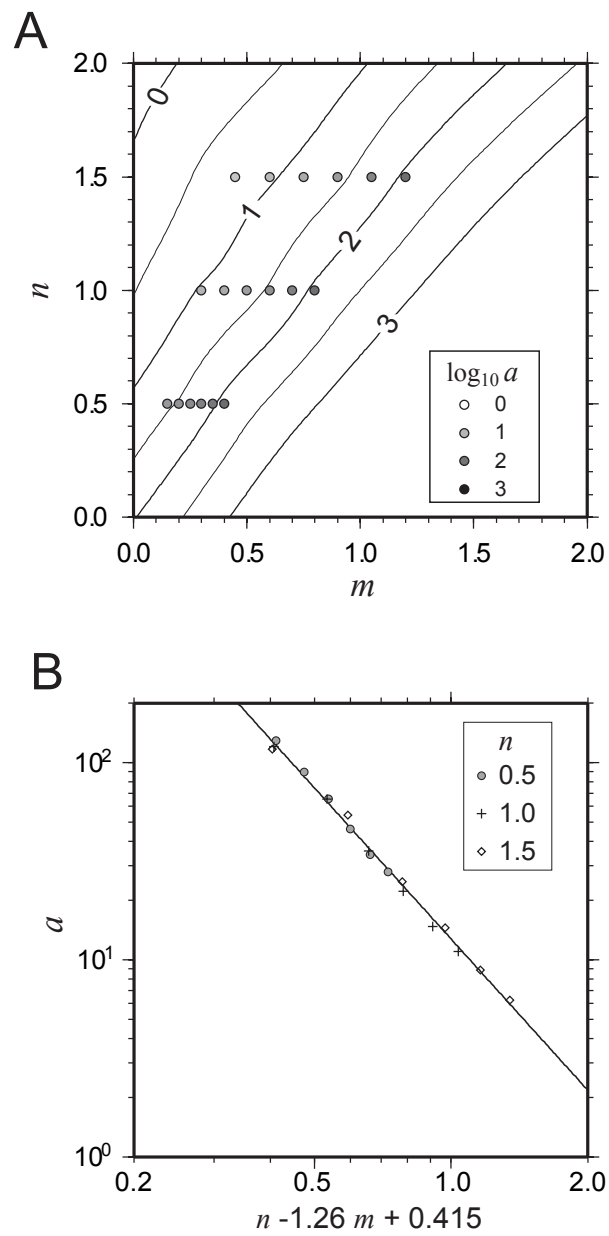


Fig. 7

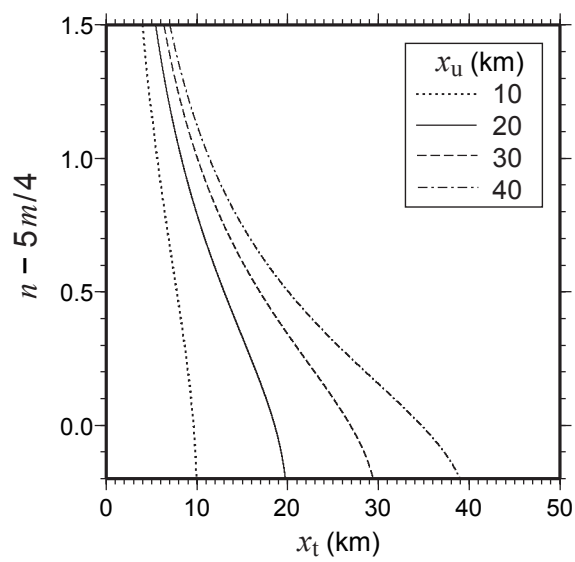


Fig. 8

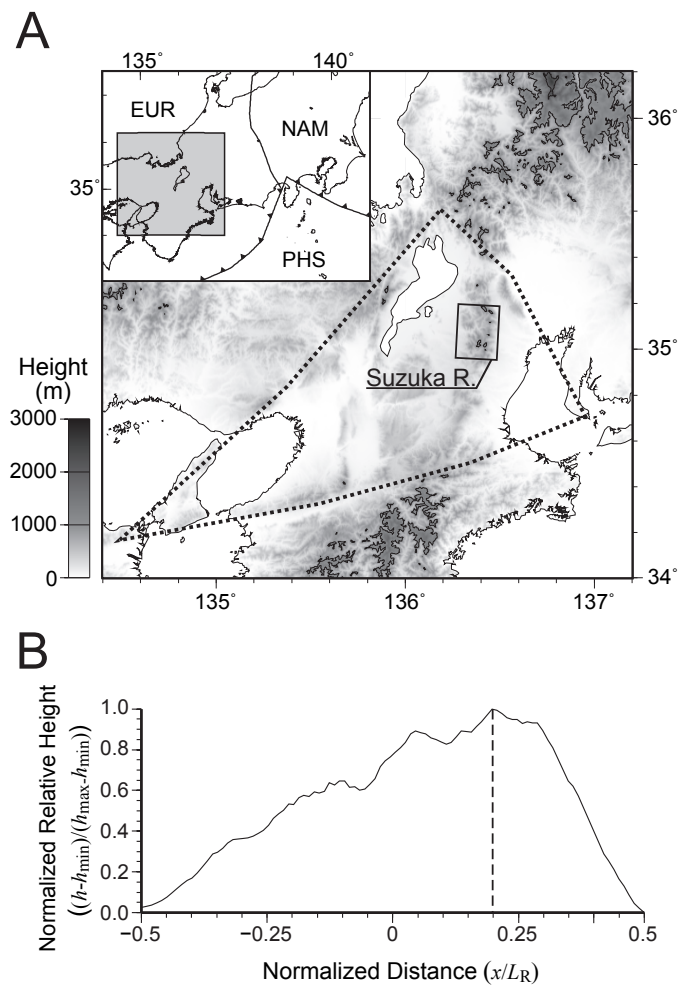


Fig. 9

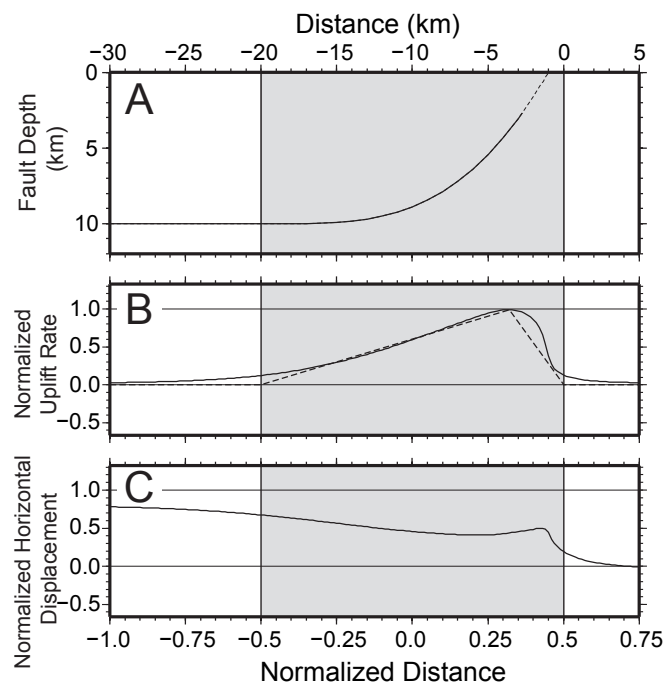


Fig. 10